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Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2019

Third Semester

Complementary Course—Mathematics

VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY

(Common For B.Sc. Physics, Chemistry, Petrochemicals Geology Computer Maintenance and Electronics and Food Science and Quality Control)

(2013—2016 Admissions)

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. State Green's theorem in plane.
2. Find the angle between the vectors $(2, 4, 4)$ and $(2, 0, 0)$.
3. State Gauss's Divergence theorem.
4. Find the curl of $\overrightarrow{F} = xyz\left(x\,\hat{i} + y\,\hat{j} + z\,\hat{k}\right)$.
5. Write the parametric representation of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
6. Write the general form of Clairaut's equation.
7. Solve : $19\frac{dy}{dx} + 90y = 0$.
8. Find the eccentricity of the ellipse $6x^2 + 8y^2 = 48$.

Turn over

9. Find the asymptotes of the hyperbola $\frac{y^2}{9} - \frac{x^2}{25} = 1$.

10. Find the integrating factor of the differential equation $\left(x^2 + y^2 + x\right) dx + xy\, dy = 0$,.

(10 × 1 = 10)

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. If f and g are differentiable functions, then prove that $\text{div}\,(f \times g) = g.\,\text{curl}\, f - f.\,\text{curl}\, g$.

12. Evaluate $\int \overline{F}.\, dr$, where $F = xy\,\hat{i} + yz^2\,\hat{j} + y^2 z\hat{k}$ from origin to the point $(1, 1, 1)$ along the curve $x = t^2; y = t^3, z = t^4$.

13. Find a unit normal vector to the surface $x^2 + y^2 + z^2 = 36$.

14. Solve : $\left(x^2 + y^2\right) dy = xydx$.

15. Solve : $\left(x^2 + y^2 + x\right) dx + xydy = 0$.

16. Solve : $\frac{xdy - ydx}{x^2 + y^2}$.

17. Find the curl of the vector field $\vec{f} = xy\hat{i} - yz\hat{j} + xz\hat{k}$.

18. Solve the differential equation $\left(x^2 + y^2\right) dx - 2xy\, dy$.

19. Find the focus and directrix of the parabola $y = -32x^2$.

20. Sketch the curve $6x^2 - 3y^2 = 1$.

21. Find the polar equation of the circle $(x-2)^2+(y-4)^2=16$.

22. Find an equation for the ellipse of eccentricity $\frac{2}{3}$ that has the line $x=9$ as a directrix and the point $(4,0)$ as the corresponding focus.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. If $\nabla\varphi=\left(x^2+yz\right)\hat{i}+\left(y^2+zx\right)\hat{j}+\left(z^2+xy\right)\hat{k}$, find φ.

24. Prove that the necessary and sufficient condition for the vector $\bar{u}(t)$ to have constant magnitude is that $\bar{u}.\frac{d\bar{u}}{dt}=0$.

25. Find the flux of $F=(x-y)\hat{i}+x\hat{j}$ across the circle $x^2+y^2=1$ in the xy-plane.

26. Find the area of the region in the first quadrant within the cardioid $r=a(1-\cos\theta)$.

27. Solve $xdy-ydx=\left(x^2+y^2\right)dx$.

28. Find the orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2=a/x$.

29. Derive the standard equation of the parabola.

30. Find a Cartesian equation for the hyperbola centered at the origin that has a focus at $(3,0)$ and the line $x=1$ as the corresponding directrix.

31. Find an equation for the circle through the points $(1,0)$, $(0,1)$, and $(2,2)$.

(6 × 4 = 24)

Turn over

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Evaluate $\iint_S A \cdot \hat{n} dS$ where $A = yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}$ and S is the region bounded by $x^2 + y^2 + z^2 = 1$, in the first octant.

(b) Find by Green's Theorem, the value of $\int_C \left(x^2\, ydx + ydy\right)$ where C is the closed curve formed by $y^2 = x$ and $y = x$ between $(0, 0)$ and $(1, 1)$.

33. (a) Find the centroid of the region that is bounded below by the x-axis and above by the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

(b) Draw an ellipse of eccentricity $\frac{4}{5}$.

34. (a) State Gauss's Divergence Theorem. Use it to evaluate $\iint_S \bar{F} \cdot \bar{n} dS$ where $\bar{F} = \left(x^2 - yz\right)\bar{i} + \left(y^2 - zx\right)\bar{j} + \left(z^3 - xy\right)\bar{k}$ over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.

(b) Prove that $\text{div}\, \phi f = \phi \text{div}\, f + f.(\text{grad}\, \phi)$ and hence show that $\text{div}\left(r^3 \bar{r}\right) = 6r^3$.

35. (a) Sketch the conic $r = \frac{6}{2 + \cos\theta}$.

(b) Find the points of intersection of the pairs of curves $r^2 = \sin 2\theta$ and $r^2 = \cos 2\theta$.

(2 × 15 = 30)