



QP CODE: 24044651



Reg No :

Name :

M.Sc DEGREE (CSS) EXAMINATION, OCTOBER 2024

Third Semester

M.Sc MATHEMATICS , M.Sc MATHEMATICS (SF)

CORE - ME010301 - ADVANCED COMPLEX ANALYSIS

2019 ADMISSION ONWARDS

AF6C7C78

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight 1 each.

1. Define a symmetric region and give an example.
2. What do you mean by the mean value property of a real valued function $u(z)$ in a region Ω ?
3. Write the Taylor's series expansions of e^z and $\sin z$ about the origin.
4. Define Canonical product.
5. Prove that $\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin\pi z}$.
6. Define Riemann zeta function. Also check whether the number of primes is finite or not.
7. Using Riemann functional equation prove $\zeta(1-s) = 2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$.
8. Where do the zeros of zeta function lie in the complex plane?
9. What is meant by the boundary behavior?
10. Prove that $\frac{\sigma'(z)}{\sigma(z)} = \zeta'(z)$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any **six** questions.

Weight 2 each.

11. State and prove maximum principle for harmonic functions.
12. State and prove Schwarz theorem.





13. Write the general form of a Laurent series for $f(z)$ which is analytic in the annulus $R_1 < |z - a| < R_2$. Derive the Laurent series of $f(z) = \frac{e^z}{(z+1)^2}$ about $z = -1$.
14. State Mittag-Leffler's theorem. Prove that $\pi \operatorname{cosec} \pi z = \lim_{n \rightarrow \infty} \sum_{n=-m}^m (-1)^n \frac{1}{z-n}$.
15. Define normal family and totally bounded family of functions. Prove that a sequence of functions converges uniformly to f on compact subsets if and only if it converges to f with respect to ρ .
16. If a family \mathcal{F} of continuous functions with values in a metric space S is normal in a region Ω of the complex plane then prove that \mathcal{F} is equicontinuous on every compact subsets of Ω .
17. Prove that a non constant elliptic function has equally many zeros and poles.
18. Prove that $\zeta(z + u) = \zeta(z) + \zeta(u) + \frac{1}{2} \frac{\wp'(z) - \wp'(u)}{\wp(z) - \wp(u)}$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. Define a subharmonic function. State and prove any three properties.
20. Obtain Jensen's formula. Deduce Poisson-Jensen formula.
21. (i) Describe the Riemann Zeta function.
(ii) Prove that the Zeta function can be extended to the whole plane as a meromorphic function having a single, simple pole at $s = 1$.
(iii) Find the residue of the Zeta function at $s = 1$.
22. (a) Prove that any simply connected region other than the complex plane is topologically equivalent to the unit disk.
(b) Prove that the Riemann mapping is unique.

(2×5=10 weightage)

