QP CODE: 25047271

25047271

Reg No : ....

Name : ....

M.Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2025

Third Semester

M Sc MATHEMATICS

Core Course - ME010301 - ADVANCED COMPLEX ANALYSIS

2019 ADMISSION ONWARDS

4A08F4D8

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define a harmonic function and give an example.
2. Show that *every harmonic function is subharmonic.*
3. Find the coefficient of z^7 in the expansion of $tanz$ as a Taylor's series.
4. Expand $f(z) = \frac{1}{z(z-1)}$ as Laurent's series in powers of z.
5. Prove that $\Gamma(z)\Gamma(1-z) = \frac{\pi}{sin\pi z}$.
6. *Where do the zeros of zeta function lie in the complex plane?*
7. *Prove that a sequence of functions in $\mathcal{F}$ converges uniformly to f on compact subsets if and only if it converges to f with respect to the distance function ρ in $\mathcal{F}$.*
8. State Arzela- Ascoli's theorem.
9. State Riemann mapping theorem.
10. Define a periodic function. Give examples of meromorphic functions which are not periodic, simply periodic and doubly periodic.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. (a) Suppose that $u(z)$ is harmonic for $|z| < R$ and continuous for $|z| \leq R$. Prove that $u(a) = \frac{1}{2\pi} \int\limits_{|z|=R} \frac{(R^2 - |a|^2)}{|z-a|^2} u(z) d\theta$, for all $|a| < R$.

(b) If $|a| < R$, then evaluate $\int_{|z|=R} \frac{(R^2-|a|^2)}{|z-a|^2} d\theta.$

12. Prove that if $f(z)$ is analytic in a region Ω, then $\overline{f(\overline{z})}$ is analytic in $\Omega^* = \{\overline{z}; z \in \Omega\}$.

13. State Mittag-Leffler's theorem. Prove that $\pi cosec \pi z = \lim_{n\to\infty} \sum_{n=-m}^{m} (-1)^n \frac{1}{z-n}$.

14. Define genus and order of an entire function and state the relationship between them. Also give example of an entire function, find its order and genus.

15. Prove that the Riemann Zeta function can be extended to a meromorphic function in the whole plane whose only pole is a simple pole at $s = 1$ with residue 1.

16. *Prove* $\xi(s)$ *is entire and* $\xi(s) = \xi(1-s)$ *where* $\xi(s) = \frac{1}{2}s(1-s)\pi^{-(\frac{s}{2})}\Gamma(\frac{s}{2})\zeta(s)$.

17. Suppose that the boundary of a simply connected region Ω contains a line segment γ as a one sided free boundary arc. Then prove that the function $f(z)$ which maps Ω onto the unit disk can be extended to a function which is analytic and one- to- one on $\Omega \cup \gamma$. Prove that the image of γ is an arc γ' on the unit circle.

18. Show that any even elliptic function with periods ω_1 and ω_2 can be written in the form $C \prod_{k=1}^{n} \frac{\wp(z) - \wp(a_k)}{\wp(z) - \wp(b_k)}$, where C is a constant.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. *(a) State and prove Harnack's inequality.*
(b) State and prove Harnack's Principle

20. Obtain the general form of an entire function with arbitrary zeros $a_n \neq 0$ with $\lim_{n\to\infty} a_n = \infty$.

21. (i) Prove that the Riemann Zeta function is analytic in the half plane $Re\ s > 1$.
(ii) Establish the connection between the Zeta function and the sequence of prime numbers.
(iii) Derive the integral representation of the Zeta function.

22. Prove that $[\wp'(z)]^2 = 4[\wp(z)]^3 - g_2\wp(z) - g_3$, where g_2 and g_3 are constants.

(2×5=10 weightage)