QP CODE: 25024356

25024356

Reg No : .....

Name : .....

M.Sc DEGREE (CSS) EXAMINATION, APRIL 2025

Fourth Semester

M Sc MATHEMATICS

ELECTIVE - ME800402 - ALGORITHMIC GRAPH THEORY

2019 ADMISSION ONWARDS

6734841D

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define order and size of a graph G. Find an upper bound for the number of edges in a (p, q) graph.
2. Define union of two graphs. Find $K_{1,3} \cup 2K_3 \cup 3K_1$.
3. What is complexity of an algorithm? When can you say that an algorithm efficient?
4. Define a tree. Draw all trees of order 5.
5. If T is a binary tree of height h and order p, then prove that $h \geq \lceil log_2(\frac{p+1}{2}) \rceil$
6. Define distance function on a graph G and on a directed graph D.
7. Define Net-flow out of a vertex and net-flow into a vertex in a network N.
8. Write a note on saturation arc of a network N with respect to the flow f and f -augmenting semipath Q .
9. *Define an alternating path and an augmenting path in a graph with example*
10. *Define a BIBD*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. (a) Define degree set of a graph. What is the degree set of a r-regular graph?
    (b) Draw a graph having the degree set $D = \{0, 4, 5\}$.

12. Draw the graph G be with vertex set $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $E(D) = \{v_1v_2, v_1v_3, v_2v_3, v_3v_4, v_3v_5\}$. Determine its adjacency matrix, adjacency list, adjacency list table

13. Explain BFS Algorithm

14. Define the minimum spanning tree problem. Write an algorithm to determine a minimum spanning tree in a non-trivial connected weighted graph whose edges have distinct weights.

15. Let G be a graph of order p and let n be an integer with $1 \leq n \leq p-1$. If $\delta(G) \geq \frac{(p+n-2)}{2}$, then prove that G is $n-$connected.

16. Prove that for $n \geq 1$, , if a graph G is $n-$ connected , then every pair of vertices of G is connected by at least n internally disjoint paths.

17. *Find a maximum weight perfect matching in the weighted complete bipartite graph G with partite sets* $\{v_1, v_2, v_3, v_4. v_5\}$ *and* $\{u_1, u_2, u_3, u_4, u_5\}$*where G is represented by the given matrix* $M = [m_{ij}]$ *with* $w(v_i, u_j) = [m_{ij}]$

$$\begin{bmatrix} 5 & 1 & 1 & 3 & 2 \\ 0 & 1 & 3 & 3 & 4 \\ 2 & 5 & 4 & 3 & 0 \\ 2 & 2 & 3 & 4 & 4 \\ 6 & 2 & 0 & 0 & 1 \end{bmatrix}$$

18. *Prove that for every positive integer n, the graph* K_{2n} *is 1-factorable*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) An edge e of a connected graph is a bridge if and only if e does not lie on any of the cycle on G.
b) Show that every $u-v$ walk in a graph contains a $u-v$ path.

20. Write critical path algorithm. Find its complexity.

21. State and prove Menger's theorem.

22. *Write the Kuhn Munkers algorithm to find a maximum weight perfect matching in a weighted complete bipartite graph G*

(2×5=10 weightage)