QP CODE: 24045337

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Reg No : ....

Name : ....

M.Sc DEGREE (CSS) EXAMINATION, DECEMBER 2024

First Semester

CORE - ME010101 - ABSTRACT ALGEBRA

M.Sc MATHEMATICS,M.Sc MATHEMATICS(SF)

2019 ADMISSION ONWARDS

9E8DEE2C

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the maximum possible order for some element of $\mathbb{Z}_4 \times \mathbb{Z}_{18} \times \mathbb{Z}_{15}$
2. *Is every group a* G *- set? Justify your answer.*
3. *Define torsion subgroup of an abelian group. Find the torsion subgroup of* $\mathbb{Z}_4 \times \mathbb{Z} \times \mathbb{Z}_3$
4. Find the kernel of the homomorphism $\phi : \mathbb{Z}_{12} \to \mathbb{Z}_3$, where $\phi(1) = 2$.
5. Prove that every group of order 159 is cyclic.
6. Prove that no group of order 45 is simple.
7. *Compute the evaluation homomorphism* $\phi_3[(x^4 + 2x)(x^3 - 3x^2 + 3)]$, $F = E = \mathbb{Z}_7$
8. *Check* $25x^5 - 9x^4 - 3x^2 - 12$ *is irreducible over* $\mathbb{Q}$
9. Define ring homomorphism and its kernel.
10. Find atleast one $c \in \mathbb{Z}_3$ such that $\mathbb{Z}_3[x]/(x^2 + c)$ is a field.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Prove that if* m *divides order of a finite abelian group* G, *then* G *has a subgroup of order* m.
12. *Show that an intersection of normal subgroups of a group* G *is again a normal subgroup of* G.
13. Let H be a p-subgroup of a finite group G. Prove that (N[H] : H) ≡ (G : H) (mod p).
14. Prove that the center of a finite nontrivial p-group G is nontrivial.
15. *Find all solutions of the congruence 22x ≡ 5 mod 15*

16. *Factorize* $x^4 + 3x^3 + 2x + 4$ *into linear factors in* $\mathbb{Z}_5[x]$

17. Let N be an ideal of a ring R. Prove that the additive cosets of N form a ring.

18. Prove that a field F is either of prime characteristic p and contains a subfield isomorphic to $\mathbb{Z}_p$ or of characteristic 0 and contains a subfield isomorphic to $\mathbb{Q}$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. (a) Let X be a G – set. Prove that G_x is a subgroup of G for each $x \in X$.
(b) *Let* X *be a* G *– set and let* $x \in X$*. Prove that* $|Gx| = (G : G_x)$*. Also prove that if* $|G|$ *is finite , then* $|Gx|$ *is a divisor of* $|G|$.

20. (a) State and prove second Sylow theorem.
(b) Let G be a finite group and let P be a normal p-subgroup of G. Show that P is contained in every Sylow p-subgroup of G.

21. *Given F be the desired field of quotients of an integral domain D. For [(a,b)] and[(c,d)] in F, the equations [(a,b)] +[(c,d)] = [(ad+bc,bd)] and [(a,b)] [(c,d)] = [(ac,bd)] give well defined binary operations. Prove that F satisfies all the field axioms.*

22. Explain Quaternion ring. Prove that Quaternion ring is a strictly skew field.

(2×5=10 weightage)