

M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2016

Second Semester

Faculty of Science

Branch I (A)—Mathematics

MT 02 C06—ABSTRACT ALGEBRA

(2012 Admissions)

Time : Three Hours

Maximum Weight : 30

Part A

*Answer any five questions.
Each question has weight 1.*

1. Find the order of $(8, 4, 10)$ in $Z_{12} \times Z_{60} \times Z_{24}$.
2. Give an example to show that a polynomial irreducible over a field need not be irreducible over a larger field.
3. Find the basis of $\mathbb{Q}(\sqrt[3]{2}, \sqrt{3})$ over \mathbb{Q} .
4. Prove or disprove : If α and β are constructible real numbers then so is α/β .
5. For a prime number p , every group G of order p^2 is abelian—Prove.
6. Explain : (i) Class equation ; (ii) Conjugate class.
7. Find the order of $G \left(\mathbb{Q}(\sqrt[3]{2})/\mathbb{Q} \right)$.
8. Prove or disprove : $\mathbb{Q}[\sqrt{2}, \sqrt{3}]$ is separable over \mathbb{Q} .

(5 × 1 = 5)

Part B

*Answer any five questions.
Each question has weight 2.*

9. Prove that a direct product of abelian groups is abelian.
10. Define the ring of polynomials $R[x]$ and verify the axioms of ring. Show also that $R[x]$ is commutative if R is commutative.

Turn over

11. Show that all finite fields must have prime power order.

12. Prove in detail $\mathbb{Q}(\sqrt{3} + \sqrt{7}) = \mathbb{Q}(\sqrt{3}, \sqrt{7})$.

13. If H and k are finite subgroups of a group G , prove :

$$|Hk| = \frac{|H||k|}{|H \cap k|}$$

14. Show that there are no simple groups of order $255 = (3)(5)(17)$.

15. Show that $f(x) \in F[x]$ has no zero of multiplicity > 1 if and only if $f(x)$ and $f'(x)$ have no common factor in $\bar{F}[x]$ of degree > 0 .

16. Show that an automorphism of a splitting field E over F of a polynomial $f(x) \in F[x]$ permutes the zeros of $f(x)$ in E .

(5 × 2 = 10)

Part C

*Answer any three questions.
Each question has weight 5.*

17. (a) Establish Eisenstein criterion .

(b) Establish uniqueness of factorization in $F[x]$.

18. (a) Show that $x^3 + 17x + 36$ is irreducible over $\mathbb{Q}[x]$.

(b) Find the number of irreducible polynomials in $Z_p[x]$ where p is prime.

19. (a) Show that the set of all constructible real numbers form a subfield of real numbers.

(b) Show algebraically that it is possible to construct an angle of 30° .

20. Let F be a field and let α and β be algebraic over F with $\deg(\alpha, F) = n$. Prove that the map $\psi_{\alpha, \beta} : F(\alpha) \rightarrow F(\beta)$ defined by

$$\psi_{\alpha, \beta} (c_0 + c_1\alpha + \dots + c_{n-1}\alpha^{n-1}) = c_0 + c_1\beta + \dots + c_{n-1}\beta^{n-1}$$

for $c_i \in F$ is an isomorphism of $F(\alpha)$ onto $F(\beta)$ iff α and β are conjugate over F .

21. State and prove Frobenius Automorphism Theorem.
22. State and prove the main theorem of Galois theory.

(3 × 5 = 15)