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**Reg. No.................................**

**Name....................................**

# M.Sc. DEGREE (C.S.S.) EXAMINATION, JUNE 2019

**Second Semester**

Faculty of Science

Branch I (a) : Mathematics

MTO 2C 06—ABSTRACT ALGEBRA

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

## Part A

*Answer any* **five** *questions.*
*Each question has weight* 1.

1. A polynomial may be irreducible over a field but may not be irreducible if viewed over a larger field—Give example.
2. Define torsion subgroup and find a torsion subgroup of $z \times z_2$.
3. A finite extension field E of a field F is an algebraic extension of F—Prove.
4. Prove : Squaring the circle is impossible.
5. State isomorphism extension theorem.
6. Obtain necessary and sufficient condition for a finite group G to be a $p$-group.
7. Obtain the splitting field of $x^3 - 2$ over Q.
8. If E is the finite extension of F, show that {E : F} divides [E : F].

(5 × 1 = 5)

## Part B

*Answer any* **five** *questions.*
*Each question has weight* 2.

9. State and prove the Lemma describing, upto isomorphism, of all finite Abelian groups.
10. State and prove division algorithm.
11. If $\alpha$ and $\beta$ are constructable real numbers, show that $\alpha\beta$ and $\alpha/\beta$ if $\beta \neq 0$ are also constructable.
12. State the required Lemma and show that a finite field GF $(p^n)$ of $p^n$ elements exists for every prime power $p^n$.
13. Obtain the order of the group $G\left(Q\left(\sqrt{2}, \sqrt{3}\right)/Q\right)$.

**Turn over**

14. Use Sylow theorems to show that no group of order IS is simple.

15. Find the degrees of the splitting fields $Q\left(\sqrt[3]{2}, i\sqrt{3}\right)$ and $Q\left(\sqrt[3]{2}, i, \sqrt{3}\right)$.

16. State the main theorem of Galois theory.

(5 × 2 = 10)

## Part C

*Answer any* **three** *questions.*
*Each question has weight* 5.

17. Obtain necessary and sufficient conditions for a group to be the internal direct product of subgroups H and K.

18. Characterise the maximal ideals of F $[x]$.

19. Establish Kronecker's theorem on extension fields. Illustrate the construction involved in the proof of the theorem by an example.

20. Define (i) A finite extension field E of a field and an algebraic extension of a field ; (ii) If E is a finite extension field of a field F and K is a finite extension field of E prove that K is a finite extension of F and $[K:F]=[K:E][E:F]$.

21. State and prove the theorem on basic isomorphism of algebraic field theory. Deduce that complex zeros of polynomials with real coefficient occur in conjugate pairs.

22. If K is a finite extension of E and E is a finite extension of F show that K is separable over F if and only if K is separable over E and E is separable over F. Also prove that if E is a finite extension of F, then E is separable over F if and only if each α in E is separable over F.

(3 × 5 = 15)