F 5068 (Pages : 3) Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JANUARY 2016

Third Semester

Faculty of Science

Branch I (A) : Mathematics

MT 03 C15—OPTIMIZATION TECHNIQUES

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. What is integer programming ? How does it differ from linear programming ?
2. How does the optimal solution of an integer programming problem compared with that of the linear programming problem.
3. What do you understand by the term sensitivity analysis ?
4. State the problem of minimum potential difference in a network.
5. Define the term zero sum game.
6. Define the terms pay-off and optimal strategies.
7. Define the terms : (a) Hessian matrix ; and (b) Gradient vector.
8. What is restricted basis entry and why it is necessary ?

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Prove that if an optimal solution of the problem :

$$\left.\begin{array}{l}\text{Minimise } f(x) = CX \\ \text{subject to } X \in S_F\end{array}\right\}$$

is an integer or a mixed integer vector $X \geq 0$, then it is also an optimal solution of the problem :

$$\left.\begin{array}{l}\text{Minimise } f(x) = CX \\ \text{subject to } X \in T_F\end{array}\right\}.$$

Turn over

10. Write a short note on Gomory's cutting plane algorithm.

11. A manufacturing firm produces two types of products A and B. The unit profit of a product A is Rs. 100 and that of product B is Rs. 50. The goal of the firm is to earn a total profit of exactly Rs. 700 in the next week. Formulate the problem as goal programming.

12. Give algorithm to find the spanning tree of minimum length.

13. Explain the difference between pure strategy and mixed strategy.

14. What is a symmetric game ? Show that the value of a symmetric game is zero.

15. Calculate the value of $f(x) = 2x^3 - 3x^2 + x - 4$ at $x = 7$ starting from the point $x_o = 3$ using Taylor's series approximation.

16. Show that the point $x = (0, 0)$ is a global minimum solution to $f(x) = x_2^2 + 3x_1^6 + 5x_2^4$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*

Each question has weight 5.

17. Solve by branch and bound method :

Maximize $x_1 + 2x_2$

subject to $5x_1 + 7x_2 \le 21$

$-x_1 + 3x_2 \le 8$

x_1, x_2 non-negative integers.

18. Solve the LP problem :

Maximize $Z = 4x_1 + 6x_2 + 2x_3$

subject to the constraints $x_1 + x_2 + x_3 \le 3$

$x_1 + 4x_2 + 7x_3 \le 9$

and $x_1, x_2, x_3 \ge 0$.

Discuss the effect of change in the availability of resources from $[3, 9]^T$ to $[9, 6]^T$.

19. State and prove minimax theorem.

20. Give the algorithm to solve a generalised problem of maximum flow.

21. Solve the problem :

Minimize $f(x) = 4x_1 - x_2^2 - 6$

subject to $26 - x_1^2 - x_2^2 = 0$

using the projected gradient method.

22. Solve, minimize $f(x) = 50 + (2.71 - x_1)^2 + (1 - x_2)^2$ start from the point $x = (0.5, 0)$. Using a Hooke-Jecues search. Start search with unit moves along the co-ordinate axis and continue until the percent change in the objective function is 2 percent or less. Plot the trajectory.

(3 × 5 = 15)