

18001761



Reg. No.....

Name.....

**M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2018**

**Third Semester**

Faculty of Science

Branch I (A)–Mathematics

MT 03 C 15—OPTIMIZATION TECHNIQUES

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

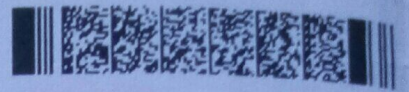
**Part A**

*Answer any five questions.  
Each question has weight 1.*

1. How does integer programming differ from linear programming ?
2. What is the effect of the 'integer' restriction of all the variables on the feasible space of integer programming problem ?
3. Define the following :
  - (a) Competitive game.
  - (b) Pure and mixed strategies.
4. Define the following :
  - (a) Pay-off matrix.
  - (b) Rectangular game.
5. Define the following :
  - (a) A chain.
  - (b) A path.
  - (c) A connected graph.
6. Write a short note on sensitivity analysis.
7. Define the following terms :
  - (a) Gradient vector.
  - (b) Hessian matrix.
8. What are the primary uses of Kuhn–Tucker necessary and sufficient conditions ?

(5 × 1 = 5)



**Part B**

Answer any **five** questions.  
Each question has weight 2.

9. Describe the branch and bound method in integer programming.
10. Solve the problem using cutting plane method :

Maximise  $x_1 + x_2$

subject to  $7x_1 - 6x_2 \leq 5$

$6x_1 + 3x_2 \geq 7$

$-3x_1 + 8x_2 \leq 6$

$x_1, x_2$  non-negative integers.

11. Five villages in a hilly region are connected by roads. The direct distance (in km) between each pair of villages along a possible road and its cost of construction per km (in  $10^4$  rupees) are given below. Find the minimum cost at which all the villages can be connected and the roads which should be constructed :

		Distances				
		1	2	3	4	5
Costs	1		18	12	15	10
	2	3		15	8	22
	3	4	3		6	20
	4	5	5	6		7
	5	2	2	5	7	

12. Write a short note on sensitivity analysis.
13. Use Graphical methods to solve the game with pay-off matrix :

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$$

14. Explain the principle of dominance in Game theory.



15. Minimize the following objective function using a Golden section search. Use a resolution of  $\epsilon = 0.10$ .

$$\text{Minimize } f(x) = 3x^4 + (x-1)^2$$

$$4 \geq x \geq 0.$$

16. Show that the point  $x = (0.0)$  is a global minimum solution to  $f(x) = x_2^2 + 3x_1^6 + 5x_2^4$ .

(5 × 2 = 10)

### Part C

Answer any **three** questions.

Each question has weight 5.

17. (a) Maximize  $5x_1 + 2x_2 + x_3$   
subject to  $x_1 + x_2 + 2x_3 \leq 10$ ,  $|-3x_1 + 10x_2 - x_3| \geq 15$ ,  $x_1, x_2, x_3 \geq 0$ .
- (b) Express the following conditions as simultaneous constraints using 0.1 variables :
- Either  $x_1 + 2x_2 \leq 4$  or  $2x_1 + 3x_2 \geq 12$ .
  - If  $x_3 \leq 4$  then  $x_4 \geq 5$ , otherwise  $x_4 \leq 2$ .
  - $x_5 = 1$  or 3 or 5 only.
  - At least two of the following constraints are satisfied  $x_6 + x_7 \leq 3$ ,  $x_6 \leq 2$ ,  $x_7 \leq 4$ ,  $x_6 + x_7 \geq 5$ .

18. State and prove Mini-max theorem.

19. Solve the following LP problem :

$$\text{Maximize } Z = 3x_1 + 5x_2$$

subject to the constraints  $3x_1 + 2x_2 \leq 18$

$$x_1 \leq 4$$

$$x_2 \leq 6, x_1, x_2 \geq 0$$

and (a) Determine the optimal solution of the problem ; (b) Discuss the change in  $c_j$  on the optimality of the optimal basic feasible solution.

20. Write both the primal and dual  $L_P$  problems corresponding to the rectangular game with the following pay-off matrix :

$$\begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix}$$

Solve the game by solving  $L_P$  problems by simplex method.



21. Solve the following problem using only the Kuhn-Tucker conditions :

$$f(x) = 100 - 1.2x_1 - 1.5x_2 + 0.3x_1^2 + 0.05x_2^2$$

$$\text{subject to } g_1(x) = x_1 + x_2 \geq 35$$

$$g_2(x) = x_1 \geq 0$$

$$g_3(x) = x_2 \geq 0.$$

22. Use projected gradient method to solve the problem :

$$\text{Minimize } f(x) = (x_1 - 3)^2 + (x_2 - 4)^2.$$

$$\text{subject to } 2x_1 + x_2 = 3.$$

(3 × 5 = 15)

