

QP CODE: 19002352



Reg No : .....

Name : .....

**M.Sc. DEGREE (C.S.S ) EXAMINATION, NOVEMBER 2019**

**First Semester**

Faculty of Science

MATHEMATICS

**Core - ME010101 - ABSTRACT ALGEBRA**

2019 Admission Onwards

7A03DA2C

Maximum Weight: 30

Time: 3 Hours

**Part A (Short Answer Questions)**

*Answer any eight questions.*

*Weight 1 each.*

1. Find all abelian groups upto isomorphism of order 720.
2. Let  $H$  be a subgroup of a group  $G$  and let  $L_H$  be the set of all left cosets of  $H$ . Prove that  $L_H$  is a  $G$ -set, where the action of  $g \in G$  on the left coset  $xH$  is given by  $g(xH) = (gx)H$ .
3. Let  $G$  be an abelian group. Let  $H$  be a subset of  $G$  consisting of the identity  $e$  together with all elements of  $G$  of order 2. Show that  $H$  is a subgroup of  $G$ .
4. State isomorphism theorems of group theory.
5. Let  $H$  be a subgroup of a group  $G$ . Then describe normalizer of  $H$  in  $G$ .
6. Let  $G$  be a finite group and let  $P$  be a normal  $p$ -subgroup of  $G$ . Show that  $P$  is contained in every Sylow  $p$ -subgroup of  $G$ .
7. Find all zeros of  $x^5 + 3x^3 + x^2 + 2x$  in  $\mathbb{Z}_5$ .
8. Check  $2x^{10} - 25x^3 + 10x^2 - 30$  is irreducible over  $\mathbb{Q}$ .
9. Define a group ring.
10. Find a prime ideal of  $\mathbb{Z} \times \mathbb{Z}$  that is not maximal.

(8×1=8 weightage)

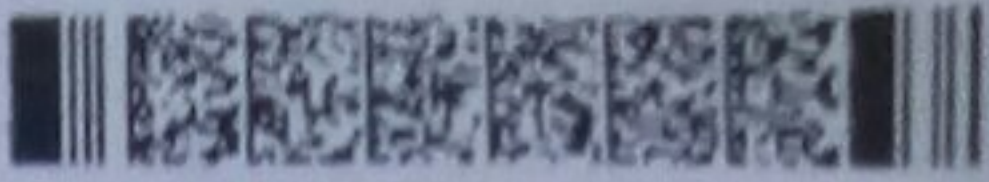
**Part B (Short Essay/Problems)**

*Answer any six questions.*

*Weight 2 each.*

11. Define conjugate subgroups. Prove that conjugacy is an equivalence relation on the collection of subgroups of a group  $G$ .



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12. Let  $X$  be a  $G$ -set. For  $x_1, x_2 \in X$ , let  $x_1 \sim x_2$  if and only if there exists  $g \in G$  such that  $gx_1 = x_2$ . Prove that  $\sim$  is an equivalence relation on  $X$ .
13. Prove that every group of order  $p^2$  is abelian, where  $p$  is a prime.
14. Prove that no group of order 48 is simple.
15. State and prove Fermat's little theorem.
16. Show that the multiplicative group of non zero elements of a finite field is cyclic.
17. Define ring homomorphism and ring of endomorphisms. Prove that the set  $\text{End}(A)$  of all endomorphisms of an abelian group  $A$  forms a ring.
18. Prove that ideal  $(p(x)) \neq \{0\}$  of  $F[x]$  is maximal if and only if  $p(x)$  is irreducible over  $F$ .

(6×2=12 weightage)

**Part C (Essay Type Questions)**

*Answer any two questions.*

*Weight 2 each.*

19. (a) Prove that a direct product of a finite number of groups forms a group.  
 (b) Prove that the group  $\mathbb{Z}_m \times \mathbb{Z}_n$  is cyclic and isomorphic to  $\mathbb{Z}_{mn}$  if and only if  $m$  and  $n$  are relatively prime.
20. (a) State and prove third Sylow theorem.  
 (b) Prove that no group of order 45 is simple.
21. Construct a field of quotients  $F$  of an integral domain  $D$  such that every element of  $F$  can be expressed as a quotient of two elements of  $D$ .
22. (a) Let  $N$  be an ideal of a ring  $R$ . Prove that the additive cosets of  $N$  form a ring.  
 (b) Let  $N$  be an ideal of a ring  $R$ . Then show that  $\gamma: R \rightarrow R/N$  defined by  $\gamma(x) = x + N$  is a ring homomorphism with kernel  $N$ .

(2×5=10 weightage)

