QP CODE: 19002352

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Reg No : ....

Name : ....

M.Sc. DEGREE (C.S.S) EXAMINATION, NOVEMBER 2019

First Semester

Faculty of Science

MATHEMATICS

Core - ME010101 - ABSTRACT ALGEBRA

2019 Admission Onwards

7A03DA2C

Maximum Weight: 30 Time: 3 Hours

Part A (Short Answer Questions)

*Answer any **eight** questions.*

Weight 1 each.

1. Find all abelian groups upto isomorphism of order 720.
2. Let H be a subgroup of a group G and let L_H be the set of all left cosets of H. Prove that L_H *is a* $G-set'$ where the action of $g \in G$ on the left coset xH is given by $g(xH) = (gx)H$.
3. Let G be an abelian group. Let H be a subset of G consisting of the identity e together with all elements of G of order 2. Show that H is a subgroup of G.
4. State isomorphism theorems of group theory.
5. Let H be a subgroup of a group G. Then describe normalizer of H in G.
6. Let G be a finite group and let P be a normal p-subgroup of G. Show that P is contained in every Sylow p-subgroup of G.
7. Find all zeros of $x^5 + 3x^3 + x^2 + 2x$ in $\mathbb{Z}_5$
8. Check $2x^{10} - 25x^3 + 10x^2 - 30$ is irreducible over $\mathbb{Q}$
9. Define a group ring.
10. Find a prime ideal of $\mathbb{Z} \times \mathbb{Z}$ that is not maximal.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

Weight 2 each.

11. Define conjugate subgroups. Prove that conjugacy is an equivalence relation on the collection of subgroups of a group G.

Turn Over

12. Let X be a G - set. For $x_1, x_2 \in X$, let $x_1 \sim x_2$ if and only if there exists $g \in G$ such that $gx_1 = x_2$. Prove that $\sim$ is an equivalence relation on X.

13. Prove that every group of order p^2 is abelian, where p is a prime.

14. Prove that no group of order 48 is simple.

15. State and prove Fermat's little theorem.

16. Show that the multiplicative group of non zero elements of a finite field is cyclic.

17. Define ring homomorphism and ring of endomorphisms. Prove that the set End(A) of all endomorphisms of an abelian group A forms a ring.

18. Prove that ideal $(p(x)) \neq \{0\}$ of F[x] is maximal if and only if $p(x)$ is irreducible over F.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any two questions.

Weight 2 each.

19. (a) Prove that a direct product of a finite number of groups forms a group.
    (b) Prove that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and isomorphic to $\mathbb{Z}_{mn}$ if and only if m and n are relatively prime.

20. (a) State and prove third Sylow theorem.
    (b) Prove that no group of order 45 is simple.

21. Construct a field of quotients F of an integral domain D such that every element of F can be expressed as a quotient of two elements of D.

22. (a) Let N be an ideal of a ring R. Prove that the additive cosets of N form a ring.
    (b) Let N be an ideal of a ring R. Then show that $\gamma : R \to R/N$ defined by $\gamma(x) = x + N$ is a ring homomorphism with kernel N.

(2×5=10 weightage)