G 3505

(Pages : 2)

Reg. No.................................

Name.....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2015

Second Semester

Faculty of Science

Branch I (a) : Mathematics

MT 02 C10—REAL ANALYSIS

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Prove : Total variation of a function over an interval is zero *if and only if* it is a constant.
2. Differentiate between Jordan arc and Jordan curve.
3. Define : Integration of Vector-Valued function.
4. Explain with example : Monotonically increasing function.
5. Explain uniform convergence of sequences with examples.
6. State Stone-Weierstrass theorem.
7. With usual notation prove $S(x)\,S(y-x) \leq 2$ where $0 < x < y$.
8. Find $\lim_{x \to 0} \frac{\log(1+x)}{x}$.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Prove : Continuous function need not be of bounded variation.
10. Examine whether $f(x) = x^2 \sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$ is a function of bounded variation. Prove.
11. If $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ *prove* $f + g \in \mathcal{R}(\alpha)$.
12. State and prove fundamental theorem of Calculus.
13. Establish the Weierstrass test for uniform convergence of sequence of functions.
14. Explain uniformly convergent sequence and point wise convergent sequence and bring out the difference between them.

Turn over

15. State and prove the theorem concerning an inversion in the order of summation of a double summation.

16. If Z is a complex number with $|Z| = 1$, there is a unique t in $[0, 2\pi)$ such that $E(it) = Z$. Prove.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) State and prove the additive property of functions of bounded variation.

(b) Characterise all rectifiable curves.

18. (a) With usual notations, state and prove the theorem to prove $\int_a^b f \, d_\alpha = \sum_1^\infty C_n f(s_n)$.

(b) If $f \in \mathcal{R}(\alpha)$ on $[a, b]$ and $C \in (a, b)$ prove $\int_a^C f \, d_\alpha + \int_c^b f \, d_\alpha = \int_a^b f \, d_\alpha$.

19. (a) State and prove change of variable theorem.

(b) State and prove the theorem to establish integration and differentiation are inverse operations.

20. (a) State and prove the theorem to establish :

$$\lim_{t \to 0} \lim_{n \to \infty} f_n(t) = \lim_{n \to \infty} \lim_{t \to 0} f_n(t).$$

(b) If $\{f_n\}$ is a sequence of continuous functions on E and if $f_n \to f$ uniformly on E, prove f is continuous on E.

21. State and prove the theorem to prove $f_n' \to f'$ if $f_n \to f$.

22. Suppose $0 < \delta < \pi$, $f(x) = 1$ if $|x| \le \delta$, $f(x) = 0$ if $\delta < |x| \le \pi$ and $f(x + 2\pi) = f(x)$ for all x. Compute the Fourier co-efficients of f an show that :

$$\sum_{n=1}^{\infty} \frac{\sin(n\delta)}{n} = \frac{\pi - \delta}{2}, \quad 0 < \delta < \pi.$$

(3 × 5 = 15)