

M.Sc. DEGREE (CSS) EXAMINATION, JANUARY 2015**Third Semester**

Faculty of Science

Branch I-(A)—Mathematics

MT 03 C15—OPTIMIZATION TECHNIQUES

(2012 admission onwards)

Time : Three Hours

Maximum Weight : 30

Part A*Answer any five questions.**Each question has weight 1.*

1. What is integer linear programming ?
2. State the 0–1 integer programming problem.
3. State the problem of maximum potential difference in a network.
4. Distinguish between :
 - (a) Path and chain.
 - (b) Cycle and circuit.
5. Define saddle point. Is it necessary that a game should always possess a saddle point.
6. Define the term pay-off matrix.
7. Define the terms :
 - (a) Hessian matrix.
 - (b) Convex set.
 - (c) Gradient vector.
8. State the necessary and sufficient conditions for a local minimum.

(5 × 1 = 5)

Turn over

Part B

Answer any **five** questions.
Each question has weight 2.

9. Show that an optimal solution of the problem :

$$\left. \begin{array}{l} \text{Minimize } f(X) = CX \\ \text{subject to } X \in TF \end{array} \right\}$$

is an optimal solution of :

$$\left. \begin{array}{l} \text{Minimize } f(X) = CX \\ \text{subject to } X \in [TF] \end{array} \right\}$$

10. Explain how cutting plane algorithm works.
11. Prove that the maximum flow in a graph is equal to the minimum of the capacities of all possible cuts in it.
12. Discuss the changes in the coefficients a_{ij} for the given linear programming problem.
Maximize $Z = CX$ subject to $AX = b, X \leq 0$.
13. Explain the difference between pure strategy and mixed strategy.
14. Examine the following pay-off matrix for saddle points :

$$\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 3 \\ 6 & 2 & 1 \end{bmatrix}$$

15. Solve the problem using constrained derivatives :

$$\text{Minimize } f(x) = 7x_1 - 6x_2 + 4x_3$$

$$\text{subject to } a_1^2 + 2x_2^2 + 3x_3^1 = 1, x_1 + 5x_2 - 3x_3 = 6.$$

16. What are the primary uses of Kuhn-Tucker necessary and sufficient conditions ?

(5 × 2 = 10)

Part C

Answer any **three** questions.
Each question has weight 5.

17. Describe the algorithm for branch and bound method for the profit maximization problem.
18. Find the maximum flow in the network with the following dataflows in arcs not necessarily being non-negative :

$$\begin{array}{l} \text{Arc} : (a,1) \quad (a,2) \quad (1,2) \quad (1,3) \quad (2,4) \quad (3,4) \quad (3,b) \quad (4,b) \\ (\text{bi},\text{ci}) : (0,10) \quad (0,5) \quad (-2,3) \quad (7,10) \quad (-3,5) \quad (-1,1) \quad (0,8) \quad (0,4) \end{array}$$

19. (a) What is goal programming ?
- (b) A factory can manufacture two products A and B. The profit on a unit of A is Rs. 80 and of B is Rs. 40. The maximum demand of A is 6 units per week, and of B is 8 units. The manufacturer has set up a goal of achieving a profit of Rs. 640 per week. Formulate the problem as goal programming and solve it. Also obtain the graphical solution.
20. Let $f(X, Y)$ be such that both $\text{Max}_X \text{Min}_Y f(X, Y)$ and $\text{Min}_Y \text{Max}_X f(X, Y)$ exist prove that a necessary and sufficient condition for the existence of a saddle point (X_0, Y_0) of $f(X, Y)$ is that $f(X_0, Y_0) = \text{Max}_X \text{Min}_Y f(X, Y) = \text{Min}_Y \text{Max}_X f(X, Y)$.
21. Describe the Hooke and Jeeves search algorithm.
22. Explain the complementary pivot algorithm to solve a complementary problem.

(3 × 5 = 15)