25900508 **Reg.No : .....**

Name : .....

MAHATMA GANDHI UNIVERSITY, KOTTAYAM

MGU-UGP (HONOURS) Regular EXAMINATION October 2025

Third SEMESTER

Discipline Specific Core Course (DSC) - MG3DSCMAT202 - ESSENTIALS OF APPLIED MATHEMATICS

(2024 ADMISSION ONWARDS)

Duration: 2 Hours **Maximum Marks: 70**

Remember(K), Understand(U), Apply(A), Analyse(An), Evaluate(E), Create(C), Skill(S), Interest(I) and Appreciation(Ap)

Students should attempt at least one question from each course outcome to enhance their overall outcome attainability.

Part A

{Short Answer Questions}

Answer any **5** questions

Each question carries **2** marks

1. Check whether $u(x, y) = 2xy$ is harmonic. [U] / [CO1]
2. Find the derivative of $f(z) = z(1 - 4z)$. [U] / [CO1]
3. Find the particular solution of the differential equation $\frac{dy}{dx} = y$ that satisfies the initial condition $y = 5$ when $x = 0$ [A] / [CO2]
4. Verify that the function $y = e^{3x}$ is a solution of the differential equation $y' - 3y = 0$. [U] / [CO2]
5. Write the polar equation of a conic with one focus at the origin and the corresponding directrix $x = k,\ k > 0$ having eccentricity e. [K] / [CO3]
6. Replace the polar equation $r^2 = 4r \sin\theta$ by equivalent Cartesian equation. [U] / [CO3]
7. Find the gradient of $f(x, y) = y - x$ at $(2, 1)$. [U] / [CO4]
8. Define radius of curvature of a plane curve. [K] / [CO4]

[2x5 = 10]

Part B

{Short Essay Questions}

Answer any **5** questions

Each question carries **6** marks

9. Determine the value of the following limits: [U] / [CO1]
   (a) $\lim_{z \to -1} \left[\frac{z+2}{z+i} \right]$ (b) $\lim_{z \to 1} \left[\frac{iz^2 - 4}{z-1} \right]$

10. Reduce $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$ to a real number. [U] / [CO1]

11. Given $z_1 = 2e^{i\pi/3}$ and $z_2 = 3e^{i\pi/6}$, find the value of the product $z_1 z_2$ in the form $a + ib$. [U] / [CO1]

12. Solve the differntial equation $y' - y \tan x = 0$ using method of separation of variables. [U] / [CO2]

13. Find the equation of the hyperbola when $\frac{y^2}{4} - \frac{x^2}{5} = 1$ is shifted 2 units down. Also find the center, foci, vertices and eccentricity of the new hyperbola. [U] / [CO3]

14. Find the vertices, foci, eccentricity and directrices of the ellipse $6x^2 + 9y^2 = 54$. [U] / [CO3]

15. The position vector of a particle at time t is given by $\vec{r}(t) = \sin t\, \hat{i} + (t^2 - \cos t)\, \hat{j} + e^t\, \hat{k}$. Find the particle's velocity, speed, direction of motion and acceleration at $t = 0$. [A] / [CO4]

16. Find the principal unit normal vector to the curve $\mathbf{r}(t) = \left(1 - 2\sin t\right)\, \hat{\imath} + \left(3 - 2\cos t\right)\, \hat{\jmath}$. [U] / [CO4]

[6x5 = 30]

Part C

{Essay Questions}

Answer any **3** questions

Each question carries **10** marks

17. (a) Find the real and imaginary parts of the function $f(z) = z^2$ both in Cartesian co-ordinates and polar co-ordinates. [U] / [CO1]
    (b) Express the function $f(z) = \frac{1}{z}$ in the form $f(z) = u(x,y) + iv(x,y)$.

18. Show that the differential equation $\frac{x}{2}\ \cot y\ \frac{dy}{dx} = -1$ is exact and solve it. [U] / [CO2]

19. (a) Solve the differential equation $y' - 2\,x\,y = x\,e^{x^2}$. [U] / [CO2]
    (b) Convert the differential equation $xy' - y = x^3$ into standard linear form and solve it.

20. Find the equation of the new parabola, when the parabola $y^2 = 8x$ is shifted vertically down by 2 units and horizontally to the right by 1 unit. Also find the focus, vertex, directrix and focal axis of the new parabola. [U] / [CO3]

21. (a) Find the length of the curve $\vec{r}(t) = (1-t)\,\hat{i} + 2t\,\hat{j} + 3t\,\hat{k}, \quad 0 \le t \le 1.$ [U] / [CO4]
(b) Find the arc length parametrization of the curve
$\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$ with base point $t_0 = 0$.

22. If $\vec{r}(t) = 2\cos t\,\hat{i} - \sin t\,\hat{j} + 4t^2\,\hat{k}$, then find the values of [U] / [CO4]
(a) $\lim_{t\to 0} \vec{r}(t)$
(b) $\lim_{t\to \frac{\pi}{2}} \vec{r}(t)$
(c) $\dfrac{d\vec{r}}{dt}$ at $t = \dfrac{\pi}{2}$.

[10x3 = 30]