

M.Sc. DEGREE (C.S.S.) EXAMINATION, AUGUST 2015**Second Semester****Faculty of Science****Branch I (a)—Mathematics****MT 02 C07—ADVANCED TOPOLOGY**

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

Part A

*Answer any five questions.
Each question has weight 1.*

1. Why Urysohn characterisation of normality theorem is remarkable ?
2. Define T_4 spaces. When they are Tychonoff ? Explain.
3. Explain : Coproduct and its summands.
4. Define cantor discontinuum.
5. What do you mean by "Filter associated with a net" ?
6. Explain the notion of sub-bases for filters.
7. State *four* equivalent condition for a T_1 topological space X to be countably compact.
8. State two results about compactness that continue to hold for countable compactness.

(5 × 1 = 5)

Part B

*Answer any five questions.
Each questions has weight 2.*

9. Let C_i be a closed subset of a space X_i for $i \in I$ show that $\prod_{i \in I} C_i$ is a closed subset of $\prod_{i \in I} X_i$ w.r. to product topology.
10. Justify the terms 'box' and 'wall' geometrically for products of copies of the red-line.
11. Characterise evaluation functions.
12. Prove : A topological space is compact iff every ultrafilter in it is convergent.

Turn over

13. Let X be a set and Y be any singleton set. Prove that \mathfrak{F} is an ultrafilter on X iff there exists function $\theta: P(X) \rightarrow P(Y)$ preserving intersections and complements such that $\mathfrak{F} = \theta^{-1}(\{Y\})$.
14. Explain :
- (i) Filter base.
 - (ii) \mathfrak{F} -neighbourhood filter.
 - (iii) Cluster point of a filter.
 - (iv) Image filter.
 - (v) Sub-filter.
15. Prove that every locally compact Hausdorff space is regular. Explain that it is not the best possible result.
16. Explain : Sequential compactness is not a productive property but it is finitely productive.

(5 × 2 = 10)

Part C

*Answer any three questions.
Each question has weight 5.*

17. State and prove Tietze characterisation of normality theorem.
18. Characterise (a) Completely regular topological space ; (b) Regular topological product ; and (c) One-to-one evaluation function.
19. State and prove Urysohn metrisation theorem.
20. (a) Characterise continuity in terms of convergence of filters.
(b) Characterise convergence of filters in topological products.
21. Establish : There is a canonical way of :
- (a) Passing from nets to filters and vice-versa.
 - (b) Bring out the relationship between filter and ultra filter.
22. Prove :
- (a) Local compactness is preserved under continuous, open functions and that it is a weakly hereditary property.
 - (b) One-point compactification of a space is Hausdorff iff the space is locally compact and Hausdorff.

(3 × 5 = 15)